Muon Spin Rotation (μSR) technique and its applications in magnetism and superconductivity

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Muon probes the local magnetism from within the unit cell

Muon probes the local magnetic response of a superconductor (Meissner screening or flux line lattice)
1. **Muon Properties**
   - Pion decay
   - Muon decay
   - Parity violation
   - Muon spin precession

2. **Muon Spin Rotation / Relaxation (µSR)**
   - Facilities around the world
   - Muon production at PSI
   - µSR instruments at PSI
   - µSR principle
   - Muon thermalization / Muon stopping sites / Muon stopping ranges
   - Measurement geometries

3. **Muon Spin Rotation / Relaxation on Magnetic Materials**
   - Different static depolarization functions and examples
   - Magnetic phase separation / coexistence of different magnetic phases
   - Magnetic fluctuations

4. **Muon Spin Rotation on Superconducting Materials**
   - Using low energy µSR to study the Meissner state of superconductors
   - Using bulk µSR to study the Vortex state of superconductors
   - Superfluid density and the symmetry of the superconducting gap
   - Magnetic and superconducting phase diagrams of Fe-based materials

5. **Summary**
Muon Properties
What is a Muon? Cosmics

Muon Flux at sea level:
~ 1 Muon/Minute/cm²

Mean Energy:
~ 2 GeV
Elementary particle/antiparticle:

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass:</td>
<td>$&lt; 200 \times$ electron mass (105.6 MeV/c$^2$) $&lt; 1/9 \times$ proton mass</td>
</tr>
<tr>
<td>charge:</td>
<td>$+e$, oder $-e$</td>
</tr>
<tr>
<td>spin:</td>
<td>$1/2$</td>
</tr>
<tr>
<td>magnetic moment:</td>
<td>$3.18 \times \mu_p$ (8.9 $\times \mu_N$), $g_H 2.00$</td>
</tr>
<tr>
<td>gyromagnetic ratio:</td>
<td>85.145 kHz/G</td>
</tr>
<tr>
<td>unstable particle:</td>
<td>mean lifetime: 2.2 $\mu$s $N(t) = N(0)\exp(-t/\tau)$</td>
</tr>
</tbody>
</table>
Muon production and polarised beams

Protons of 600 to 800 MeV kinetic energy interact with protons or neutrons of the nuclei of a light element target to produce pions.

\[ p + p \rightarrow \pi^+ + p + n \]

Pions are unstable (lifetime 26 ns). They decay into muons (and neutrinos):

\[ \pi^+ \rightarrow \mu^+ + \nu_\mu \]

The muon beam is 100 polarised with \( S_\mu \) antiparallel to \( P_\mu \).

Momentum: \( P_\mu = 29.79 \text{ MeV/c} \)

Kinetic energy: \( E_\mu = 4.12 \text{ MeV} \)
**Muon as Result of Pion Decay**

Two-body decay ➤ muon has always the energy 4.1 MeV in the reference frame of the pion (assuming $m_\mu = 0$)

Spin pion = 0 ➤ Muon has a spin 1/2 and is 100% polarized (as only left-handed neutrinos are produced)
Muon Decay

Three-body decay
Weak-decay of muon

Distribution of positrons energies
Parity-violation leading to positrons emitted anisotropically
Measuring $P(t)$: Muon Decay $\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$

- Muon decay (life time 2.2 $\mu$s) violates parity conservation → asymmetric decay
- Positrons preferentially emitted along muon spin (along polarization vector of muon ensemble)

\[
\frac{dN_{e^+}(\theta)}{d\Omega} \propto (1 + \frac{1}{3} P \cos \theta) = (1 + \frac{1}{3} P \cdot \hat{n})
\]
\[\hat{n}: \text{direction of observation (detector position)}\]

- Measuring positrons allows to observe time evolution of the polarization $P(t)$ of the muon ensemble
- Positron intensity as a function of time after implantation:

\[
N_{e^+}(t) - N_0 \left[1 + A_0 P(t) \right] e^{-\frac{t}{\tau}}
\]
\[P(t) = \bar{P}(t) \cdot \hat{n}\]

- $A_0$: Maximum observable asymmetry
  - theoretically: $A_0 = 1/3$
  - practically it depends on setup (average over solid angle, absorption in materials): $A_0 = 0.25 - 0.30$
- $A_0 P(t)$ is called asymmetry: $A(t)$

\[
\frac{dN_{e^+}(\theta)}{d\Omega} \propto (1 + \frac{1}{3} P \cos \theta)
\]
\[\theta: \text{angle between spin (polarization) and positron direction}\]
Anisotropic Muon Decay

Angular distribution of positrons from the parity violating muon decay:

\[ W(E, \theta) = 1 + a(E) \cos(\theta) \]

The asymmetry parameter \( a = 1/3 \) when all positron energies \( E \) are sampled with equal probability.

Positrons preferentially emitted along direction of muon spin at decay time

By detecting the spatial positron emission as a function of time

- time evolution of muon spin !!
Muon Spin Precession – Larmor frequency

\[ |\psi\rangle = |\uparrow\rangle \]

\[ \chi = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \]

- Field axis: quantization axis
- Spin-state eigenvalue of \( S_z \)
- Stationary state

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Muon Spin Precession – Larmor frequency

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- Field axis: quantization axis
- Spin state eigenvalue of \( S_z \)
- Stationary state

Quantum Mechanics:

- \( B_\mu \): quantization axis
- \( \chi \) in the new base \( \Rightarrow \chi = \begin{pmatrix} \cos \frac{\theta}{2} \\ \sin \frac{\theta}{2} \end{pmatrix} \)
- Calculation of \( \chi(t) \) with time dependent Schrödinger Eq.
- Calculation expectation values of \( S_\alpha \)
  \[ \langle S_\alpha \rangle = \chi^\dagger S_\alpha \chi \]
- Project back to the ref. frame of the lab.

Classically:

Torque:
\[ \tau = m \times B_\mu = \gamma \mu B_\mu \]

Euler’s Eq.:
\[ \frac{dS}{dt} = \gamma \mu S \times B_\mu \]
Muon Spin Precession – Larmor frequency

\[ P = \frac{\langle S \rangle}{\hbar/2} \]

Larmor precessions with angular velocity:
\[ \omega_L = \gamma_\mu B_\mu \]

with \( \gamma_\mu = \frac{e}{2m_\mu} g_\mu = 8.51615 \times 10^8 \text{ rad/sT} \)

Frequency:
\[ \frac{\gamma_\mu}{2\pi} = 135.539 \text{ MHz/T} \]

Classically:
Torque:
\[ \tau = m \times B_\mu = \gamma_\mu S \times B_\mu \]
Euler's Eq.:
\[ \frac{dS}{dt} = \tau \]
\[ \Rightarrow \frac{dS}{dt} = \gamma_\mu S \times B_\mu \]
**Muon Spin Precession - Bloch Equation**

**Ehrenfest Theorem**

\[ \hat{B} = \gamma \hat{m} \cdot \vec{B} \]

**Rabi Oscillations**

\[ \langle S_x \rangle \text{ and } \langle S_y \rangle \text{ precess with the Larmor frequency } \omega_L \]

**Larmor frequency**

\[ \omega_L = \gamma |\vec{B}| \]
The three key properties making SR possible:

1. The muon is 100% spin polarized.
2. The decay positron is preferentially emitted along the muon spin direction.
3. The muon spin precesses in a magnetic field.
Muon Spin Rotation / Relaxation
Facilities under study in South Corea, China, US

Generation of polarized muons ($\mu^+$)

2.2 mA $\equiv 1.4 \times 10^{16}$ Protons/sec
with 600 MeV

$p + C \rightarrow \pi^+ \pi^- p n ...$

```
~10^7 - 10^8 \mu^+/sec
100 \% pol.
~ 4 MeV
generally used for "bulk"
condensed matter studies
For thin film studies: eV-30 keV
```
Muon Production at PSI

600 MeV Proton Cyclotron at PSI:

Muons from Pion Decay:

\[ \pi^- \rightarrow \mu^+ + \nu_\mu \]
\[ \pi^- \rightarrow \mu^- + \bar{\nu}_\mu \]

Muon kinetic energy: 4 MeV
Target M
Thin graphite (5 mm)

Target E
4cm graphite

“necessary component to expand proton beam before SINQ”
**High Field \(\mu\)SR**

Muon energy: 4.2 MeV (\(\mu^+\))

9.5T, 20mK

**GPS**

*General Purpose Surface Muon Instrument*

Muon energy: 4.2 MeV (\(\mu^+\))

0.6T, 1.8K

**Shared Beam Surface Muon Facility (Muon On REquest)**

**LTF**

*Low Temperature Facility*

Muon energy: 4.2 MeV (\(\mu^+\))

3T, 20mK-4K

**LEM**

Low-energy muon beam and instrument, tunable energy (0.5-30 keV, \(\mu^+\)), thin-film, near-surface and multi-layer studies (1-300 nm)

0.3T, 2.9K

**DOLLY**

*General Purpose Surface Muon Instrument*

Muon energy: 4.2 MeV (\(\mu^+\))

0.5T, 300mK

**GPD**

*General Purpose Decay Channel Instrument*

Muon energy: 5-60 MeV (\(\mu^+\) or \(\mu^-\))

0.5T, 300mK

2.8GPa

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Principle of a SR Experiment

Implantation of muons into the sample

Muon

Sample

Mass: $m \equiv \frac{207}{9} m_e$

Magnetic moment: $3 \mu_p$

Charge: $+e$

Lifetime: $t \equiv 2.2 s$

Polarisation: $100\%$

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Interaction of the muon spin with the magnetic environment

Anisotropic muon decay in the direction of the muon spin

Detection of the decay positron

Principle of a $\mu$SR Experiment

Detection of the decay positron

Sample

$\mu$SR

Static field distribution
Sensitivity: $10^{-3} - 10^{-4}$ $B$

Positron-Detector

Positron

Fluctuation rates
(10$^5$ – 10$^9$ Hz)

$2 \mu$s
Muons stopping in matter:

4.1-MeV $\mu^+$, $v \sim 0.27 \cdot c$

$\downarrow$

ionization of atoms, $10^5$-$10^6$ excess e$^-$

$\downarrow$

2 – 3 keV, $v \sim 0.007 \cdot c$

$\downarrow$

Muonium formation ($\mu^+ e^-$), successive

e$^-$ capture and loss

$\downarrow$

100 eV, $v \sim 0.0013 \cdot c$

$\downarrow$

mainly elastic collisions, stop at intersitial site

- Total stopping time in condensed matter: $< 0.1$ ns
- Only electrostatic processes ➞ no loss of polarization
- Penetration: $\sim 0.1$ mm
SR – Muons Stopping in Matter

Muon Production

Surface Muons
Energy: 4.1 MeV
\( v \approx 0.27 \text{ c} \)

Sample

Ionization of atoms

Muonium formation (\( \mu^+ + e^- \))
charge cycling

10^5 – 10^6 excess e^-
2-3 keV
\( v \approx 0.007 \text{ c} \)

100 eV
\( v \approx 0.0013 \text{ c} \)

mainly elastic collisions
(dissociation, \( +e^- \rightarrow + \))
stop at interstitial site

thermalized \( + \)
stopping at an interstitial site

• Total stopping time in matter < 0.1 ns
• Only electrostatic processes \( \rightarrow \) no loss of polarization!
• Penetration \( \approx 0.1 \text{ mm} \)
Positive muon likes to stop:

- In the potential minimum
- High symmetry sites
- Near negative ions (e.g. $O^{2-}$, $As^{3-}$) (muon hydrogen bond like in OH with ~1 A bond length)
- Large spaces in the crystal structure
Muon Implantation Depth

Bulk $\mu$SR:
- "Normal" samples (sub-mm)
- Bulky samples + samples in containers or pressure cells

LE-$\mu$SR:
- Depth-selective investigations (1–200 nm)
\[ N(t) = Bkg + N_0 \exp\left(-t/\tau_\mu\right) \]

\( \hat{n} \): direction of detector

\( B_\mu \): internal or external field

Spin-polarized muon beam

Electronic clock

Positron detector

Counts

Time (microsec)
\[ N(t) = B_{kg} + N_0 \exp(-t/\tau_{\mu}) \left[ 1 + a \mathbf{\hat{n}} \cdot \mathbf{P}(t) \right] \]

\[ f(B_{\mu}) \]

\[ \langle B_{\mu} \rangle \]

**Frequency**: Value of field at muon site

\[ \langle L \rangle = \mathcal{B}_m B_m \]

**Damping**: Field distribution and/or dynamics
angle between magnetic field and muon polarization at $t = 0$

$P_z(t) = \cos^2 \theta + \sin^2 \theta \cos(\gamma \mu B \mu t) \int dB_\mu$

- Static part
- Oscillating part
Different Measurement Geometries

ZF and LF: Zero field and Longitudinal Field geometry

Typically used for the study of:

- Static magnetism
  - Temperature dependence of the magnetic order parameter
  - Determination of the magnetic transition temperature
  - Homogeneity of the sample
- Dynamic magnetism
  - Determination of magnetic fluctuation rates
  - Slowing down of fluctuations near phases transitions
Different Measurement Geometries

TF: Transverse Field geometry

Typically used for the study of:

- Magnetism
  - Determination of the magnetic transition temperature
  - Homogeneity of the sample
  - Knight shift (local susceptibility)
- Superconductivity
  - Absolute value and temperature dependence of the London penetration depth
  - Coherence length, vortex structure, vortex dynamics, …
Local field in magnetic materials

Internal field: generally sum of dipolar:

$$\vec{B}_{\text{dip}}(\vec{r}) = \frac{-\mu_0}{4\pi} \frac{3(\vec{\mu}_i \cdot \vec{r}_{i\mu}) \cdot \vec{r}_{i\mu} - \vec{\mu}_i r_{i\mu}^2}{r_{i\mu}^5}$$

$$B_{\text{dip}} \approx \frac{\mu_0}{4\pi} \frac{\mu_i}{r_{i\mu}^3} \approx \frac{\mu_i}{d^3} \frac{\mu_B}{A^3} \approx 0.1 \text{T}$$

and contact field (spin density at muon site):

$$\vec{B}_{\text{hf}}(\vec{r}_{i\mu}) \approx \frac{2\mu_0}{3} \mu_B \rho_{\text{spin}}(\vec{r}_{i\mu}) \approx \frac{2\mu_0}{3} \mu_B \left| \phi(\vec{r}_{i\mu}) \right|^2 < \vec{r}_{i\mu} >$$

High sensitivity:

$\mu$SR time window 10-20 μs

$\nu_\mu \approx 50 \text{ kHz}$ detectable

$B = \frac{2\pi}{\gamma_\mu} \nu_\mu \approx 0.1 \text{ mT} (\text{Gauss})$

(corresponds to 0.001μ_B or nuclear moments μ_n)
The µSR technique has a unique time window for the study of magnetic fluctuations in materials that is complementary to other experimental techniques.
Muon Spin Rotation / Relaxation on Magnetic Materials
The interesting property of magnetically ordered system is the size and temperature dependence of the magnetic moment.

How do you measure this?
Macroscopic techniques (average over the hole sample):

SQUID, PPMS, …
The interesting property of magnetically ordered system is the size and temperature dependence of the magnetic moment.

How do you measure this?
Macroscopic techniques (average over the hole sample):

- **Paramagnetism**
  - Fluctuating: $T>T_C$

- **Ferromagnetism**
  - Static: $T<T_C$

- **Antiferromagnetism**
  - Static: $T<T_N$
Magnetism

Scattering techniques: (neutrons, X-rays)

Local probes: (SR, NMR, …)

Strength of muon spin rotation / relaxation:
Investigation of magnetically inhomogeneous materials:

- Chemical inhomogeneity ("dirty samples")
- Competing interactions, coexistence of different magnetic orders, short range order, magnetic frustration
- Magnetism and superconductivity (competition and coexistence)
Magnetically Inhomogeneous Materials

Homogen:

\[ M_{\text{hom}} \]

Inhomogeneous:

\[ M_{\text{inhom}} = M_{\text{hom}} \]

Amplitude = Magnetic volume fraction
Frequency = Size of the magnetic moments (order parameter)
Damping = Inhomogeneity within the magnetic areas
Internal field at the muon site:

\[ \mathbf{B}_\mu = \mathbf{B}_c + \mathbf{B}_{dip} \]

- Contact field \( \propto e |\psi(\mathbf{r}_\mu)|^2 \)
- Dipolar contribution

\[ \mathbf{B}_{dip} = \sum_i \frac{1}{r_i^3} \left[ \frac{(3\mathbf{m}_i \cdot \mathbf{r}_i)}{r_i^2} \mathbf{r}_i - \mathbf{m}_i \right] \]

\[ B_{dip} \approx \frac{m}{r^3} \]

For \( m = 1 \mu_B \) and \( r = 1 \text{Å} \) \( \Rightarrow B_{dip} \approx 1 \text{T} \)

- Static moments as low as 0.001 \( B \) can be detected by \( \mu \text{SR} \)

- \( \mu \text{SR} \) time window: 10-20 \( \mu \)sec

- Frequencies down to 50 kHz detectable
  Fields of few Gauss (10\(^{-4}\) T)
Advantages of µSR

Muons are purely magnetic probes (I = ½, no quadrupolar effects).

Local information, interstitial probe complementary to NMR.

Large magnetic moment: \( \mu = 3.18 \mu_p = 8.89 \mu_n \) sensitive probe.

Particularly suitable for:
- Very weak effects, small moment magnetism \( \sim 10^{-3} \mu_B/\text{Atom} \)
- Random magnetism (e.g. spin glasses).
- Short range order (where neutron scattering is not sensitive).
- Independent determination of magnetic moment and of magnetic volume fraction.

Determination of magnetic/non magnetic/superconducting fractions.

Full polarization in zero field, independent of temperature unique measurements without disturbance of the system.
Different Depolarization Functions

- nuclear moments
- electronic moments
- static moments
- fluctuating moments
- various spin structures
- spin glasses (randomness)
- variety of field distributions
Magnetism of Single Crystals
Simple Magnetic Sample – Single Crystal

\[ P_z(t) = \int f(B_\mu) \left[ \cos^2 \theta + \sin^2 \theta \cos(\gamma_\mu B_\mu t) \right] dB_\mu \]

Single Crystal with \( \gamma = \pi / 2 \)

\[ P_z(t) = \cos(\gamma_\mu B_\mu t) \]

Single Crystal with \( \gamma \neq \pi / 2 \)

\[ P_z(t) = \cos^2 \theta + \sin^2 \theta \cos(\gamma_\mu B_\mu t) \]


In a single crystal the amplitude of the oscillatory component indicates the direction of the internal field.

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Example:
Single Crystalline UGe$_2$
Two magnetically inequivalent muon stopping sites

No oscillations for \( P(0) \) pointing along \( a \)-axis

\( P \) Internal fields parallel to \( a \)-axis \( (\theta = 0) \)

Oscillations around zero \( P \) because single crystal

Magnetism of Polycrystals (Powders)
\[ P_z(t) = \int f(B_\mu) \left[ \cos^2 \theta + \sin^2 \theta \cos(\gamma_\mu B_\mu t) \right] dB_\mu \]

If isotropic:

\[ f(|B_\mu|) = f(B_\mu) 4\pi B_\mu^2 \]

\[ f(B_\mu) dB_\mu = \frac{f(|B_\mu|)}{4\pi} \sin(\theta) d\theta d\phi dB_\mu \]

\[ P_z(t) = \frac{1}{3} + \frac{2}{3} \cos(\gamma_\mu B_\mu t) \]
Simple Magnetic Sample – Polycrystal

\[ P_z(t) = \int f(B_\mu) \left[ \cos^2 \theta + \sin^2 \theta \cos(\gamma_\mu B_\mu t) \right] dB_\mu \]

Polycrystal: Ideal Case

\[ f(|B_\mu|) \]

\[ B_\mu \]

\[ |B_\mu| \]

\[ P_z(t) = \frac{1}{3} + \frac{2}{3} \cos(\gamma_\mu B_\mu t) \]
Simple Magnetic Sample – Polycrystal

Polycrystal: Ideal Case

\[
P_z(t) = \frac{1}{3} + \frac{2}{3} \cos(\gamma_\mu B_\mu t)
\]

Polycrystal: Real Case

\[
f(|B_\mu|) = \frac{1}{\sqrt{2\pi \langle \Delta B_\mu^2 \rangle}} \exp\left[-\frac{(B_\mu - \langle B_\mu \rangle)^2}{2 \langle \Delta B_\mu^2 \rangle}\right]
\]

\[
\langle \Delta B_\mu^2 \rangle = \int (B_\mu - \langle B_\mu \rangle)^2 f(|B_\mu|) dB_\mu
\]

\[
P_z(t) = \frac{1}{3} + \frac{2}{3} \exp\left[-\frac{1}{2} \gamma_\mu \langle \Delta B_\mu^2 \rangle t^2\right] \cos(\gamma_\mu \langle B_\mu \rangle t)
\]
Example:
Magnetism of Polycrystalline EuFe$_2$(As$_{1-x}$P$_x$)$_2$
$A\text{Fe}_2\text{As}_2 (A=\text{Ba}, \text{Sr}, \text{Ca}, \text{Eu})$

$\text{Eu}^{2+}$ $\quad 4f^7$, $S = 7/2$

$T_{\text{SDW}}(\text{Fe}) = 190 \text{ K}$

$T_{\text{AFM}}(\text{Eu}^{2+}) = 19 \text{ K}$

Guguchia et al., PRB 83, 144516 (2011).
Magnetic structure of $\text{EuFe}_2\text{As}_2$ at 2.5 K

$A\text{Fe}_2\text{As}_2 (A=\text{Ba}, \text{Sr}, \text{Ca}, \text{Eu})$

$\text{Eu}^{2+} \rightarrow 4f^7, S = 7/2$

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Y. Xiao et al., PRB 80, 174424 (2009).

$T_{SDW}(Fe) = 190 \text{ K}$

$T_{AFM}(Eu^{2+}) = 19 \text{ K}$

Y. Xiao et al., PRB 80, 174424 (2009).


Temperature-pressure phase diagram for EuFe$_2$(As$_{0.88}$P$_{0.12}$)$_2$.

Temperature-doping phase diagram

Temperature-pressure phase diagram

Example:
Magnetism of Polycrystalline $\text{La}_{1.875}\text{Ba}_{0.125}\text{CuO}_4$
OIE effect on $T_{so}$ and magnetic fraction $V_m$

Example:
Magnetism of Polycrystalline LaOFeAs
Polycrystalline LaOFeAs

- Zero Field Muon Spin Rotation
- Static commensurate magnetic order

Mössbauer spectroscopy
Hyperfine field $B_{hf}(0) = 4.86(5)$ T
Saturation moment $= 0.25 - 0.32$

Neutron scattering
Saturation moment $= 0.36(5)$
Polycrystalline LaOFeAs

- Zero Field Muon Spin Rotation
  - Static commensurate magnetic order
  - T-dependence of the Fe magnetization with high precision, $T_N = 138$ K
  - 100% magnetic volume fraction

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Polycrystalline LaOFeAs

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Polycrystalline LaOFeAs

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- **Mössbauer spectroscopy**
  - Hyperfine field $B_{hf}(0) = 4.86(5)$ T

  \[ B_{hf} = B_{dip} + B_c + \ldots \]

  \[ B_{dip} = \sum_{i,j} \frac{1}{3} \frac{(3m_i \cdot r_i)}{r_i^2} r_i - m_i \]

  \[ \propto e |\Psi(x_{\mu})|^2 \]

  Saturation moment $\int = 0.25 - 0.32 \int_B$
**Polycrystalline LaOFeAs**

- **Zero Field Muon Spin Rotation**
  - Static commensurate magnetic order
  - $T$-dependence of the Fe magnetization with high precision, $T_N = 138$ K
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  - Hyperfine field $B_{hf}(0) = 4.86(5)$ T
  - Saturation moment $\mu = 0.25 - 0.32$ \(B\)

- **Neutron scattering**
  - Saturation moment $\mu = 0.36(5) \ B$

\[
B_{\mu} = B_{\text{dip}} + B_{c} + \cdots
\]

\[
B_{\text{dip}} = \sum_{i} \frac{1}{r_{i}^{3}} \left[ \frac{(3m_{i} \cdot r_{i})}{r_{i}^{2}} r_{i} - m_{i} \right]
\]

C. de la Cruz et al., Nature 453, 899 (2008)
Randomly Oriented Magnetic Moments

- Short Range Magnetism
- Magnetic Disorder
Randomly Oriented Moments

\[ f(B_{\mu,\alpha}) = \frac{1}{\sqrt{2\pi(\Delta B_{\mu})}} \exp\left[-\frac{B_{\mu,\alpha}^2}{2(\Delta B_{\mu})^2}\right] \]

If isotropic:
\[ f(|B_{\mu}|) = f(B_{\mu}) \cdot 4\pi B_{\mu}^2 \]

Maxwell distribution:
\[ f(|B_{\mu}|) = \frac{1}{\sqrt{2\pi(\Delta B_{\mu})^3}} \cdot 4\pi B_{\mu}^2 \exp\left[-\frac{B_{\mu}^2}{2(\Delta B_{\mu})^2}\right] \]
Randomly Oriented Moments

\[ P_z(t) = \int f(B_\mu) \left[ \cos^2 \theta + \sin^2 \theta \cos(\gamma_\mu B_\mu t) \right] d\mu \]

**Kubo-Toyabe function**

\[ P_z(t) = \frac{1}{3} + \frac{2}{3} \left[ 1 - \gamma_\mu^2 (\Delta B_\mu^2) t^2 \right] \exp \left[ -\frac{\gamma_\mu^2 (\Delta B_\mu^2) t^2}{2} \right] \]
Example:
Kubo-Toyabe depolarization due to nuclear moments InN and MnSi
InN & MnSi

**InN**
Semiconductor
Study of the hydrogen-related defect chemistry


**MnSi**
system lacking inversion symmetry
itinerant-electron magnet MnSi


---

In the paramagnetic state, a KT function is very often observed reflecting the small field distribution created by the nuclear moments
Detection of Magnetic Phase Separation
- Coexistence of Different Magnetic Phases
Example:
Helical magnetic order in MnAs
At ambient pressure CrAs is 100% magnetic!

\[ P_z(t) = \frac{1}{3} + \frac{2}{3} \cos(\gamma \mu B_\mu t) \]
Helical magnetic order

\[ P(B) = \frac{2}{\pi} \frac{B}{\sqrt{(B^2 - B^2_{\text{min}})(B^2_{\text{max}} - B^2)}} \]
Confirmation of helical type of magnetic order in CrAs
Example:
Microscopic Coexistence of Superconductivity and Magnetism in Fe-based superconductors
RuSr$_2$GdCu$_2$O$_8$

Resistivity:  
(superconductivity)

Magnetization:  
(ferromagnetism)

\[ T_{\text{Curie}} \]

\[ M_\text{[emu/mol]} \]

\[ \text{Sr} \]
\[ \text{O} \]
\[ \text{Cu} \]
\[ \text{Gd} \]
\[ \text{Ru} \]

~100%  
Magnetic volume

Mikroscopic coexistence of superconductivity and magnetism

Structure:  


Zurab Guguchia
**Static Magnetism Probed by ZF µSR**

**Single Crystals – Magnet**

\[ P_z(t) = \exp\left[ -\frac{1}{2} \gamma_\mu^2 (\Delta B_\mu^2) t^2 \right] \cos(\gamma_\mu B_\mu t) \]

- Frequency: Size of the magnetic moments
- Damping: Inhomogeneity
- Amplitude: Magnetic volume fraction

**Polycrystals – Magnet**

\[ P_z(t) = \frac{1}{3} + \frac{2}{3} \exp\left[ -\frac{1}{2} \gamma_\mu^2 (\Delta B_\mu^2) t^2 \right] \cos(\gamma_\mu \langle B_\mu \rangle t) \]

**Randomly oriented static moments**

\[ P_z(t) = \frac{1}{3} + \frac{2}{3} \left[ 1 - \gamma_\mu^2 (\Delta B_\mu^2) t^2 \right] \exp\left[ -\frac{\gamma_\mu^2 (\Delta B_\mu^2) t^2}{2} \right] \]

Zurab Guguchia
Muon Spin Rotation on Superconducting Materials
Superconductivity -- Introduction

Discovery by Kamerlingh Onnes in 1911 in mercury

Received the Nobel Prize in 1913 for “his investigations on the properties of matter at low temperatures which led, inter alia, to the production of liquid helium”. 

Temperature dependence of the resistance of a Hg sample
Superconductivity -- Introduction
<table>
<thead>
<tr>
<th>Year</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>1908</td>
<td>Kammerling Onnes: production of liquid helium</td>
</tr>
<tr>
<td>1911</td>
<td>Kammerling Onnes: discovery of zero resistance</td>
</tr>
<tr>
<td>1933</td>
<td>Meissner and Ochsenfeld: superconductors expell applied magnetic fields (MOE)</td>
</tr>
<tr>
<td>1935</td>
<td>F. and H. London: MOE is a consequence of the minimization of the electromagnetic free energy carried by superconducting current</td>
</tr>
<tr>
<td>1950</td>
<td>Ginzburg and Landau: phenomenological theory of superconductors</td>
</tr>
<tr>
<td>1950</td>
<td>Maxwell and Reynolds et al.: isotope effect</td>
</tr>
<tr>
<td>1957</td>
<td>Abrikosov: 2 types of superconductors (magnetic flux)</td>
</tr>
<tr>
<td>1957</td>
<td>Bardeen, Cooper, and Schrieffer: BCS theory -- superconducting current as a superfluid of Cooper pairs</td>
</tr>
<tr>
<td>1962</td>
<td>Josephson: Joesephson effect</td>
</tr>
<tr>
<td>1986</td>
<td>Berndoroz and Müller: High-Tc's superconductors</td>
</tr>
</tbody>
</table>
Main Characteristics:

Is a superconductor “just” an ideal conductor? New thermodynamic state of matter!

Kamerlingh Onnes

Meissner and Ochsenfeld
**Ideal Conductor**

An ideal conductor in magnetic field

**Lenz-Faraday’s law:** currents to keep $B$ constant inside of the sample

**Superconductor**

A superconductor in magnetic field

**New thermodynamic state of matter**

*From: Lecture on Superconductivity, Alexey Ustinov, Uni. Erlangen, 2007*
Nanometer scale parameters

Magnetic penetration depth: 
Coherence length:
Phenomenological London’s equations

In a sample without resistance, the electrons will feel a force:

$$\mathbf{F} = -e\mathbf{E} = m \frac{\partial(\mathbf{v})}{\partial t}$$

Recalling that the current density is: $$\mathbf{j} = -n_s e(\mathbf{v})$$

one obtains the first London equation (acceleration equation):

$$\lambda \frac{\partial \mathbf{j}}{\partial t} = \mathbf{E} \quad \text{with} \quad \lambda = \frac{m}{n_s e^2}$$

Taking the curl of this equation using the 3rd and 4th Maxwell equations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} \quad \text{(assuming} \quad \frac{\partial \mathbf{E}}{\partial t} = 0)$$

one obtains:

$$\Delta \frac{\partial \mathbf{B}}{\partial t} = \frac{1}{\lambda^2} \frac{\partial \mathbf{B}}{\partial t}$$

$$\Delta \frac{\partial \mathbf{j}}{\partial t} = \frac{1}{\lambda^2} \frac{\partial \mathbf{j}}{\partial t}$$

with: $$\lambda^2 = \frac{\Lambda}{\mu_0} = \frac{m}{\mu_0 \varepsilon^2 n_s}$$

$\Delta B = \frac{1}{\chi^2} B$

$\Delta j = \frac{1}{\chi^2} j$

$B_z(x) = B(0) \exp\left(-\frac{x}{\lambda}\right)$

$j_z(x) = \frac{I}{2\pi R \lambda} \exp\left(-\frac{x}{\lambda}\right)$

$\lambda = \sqrt{\frac{\pi \hbar^2}{\mu_0 c^2 \mu_s}}$
Ginzburg-Landau Equations (1950)

Powerful phenomenological theory, based on the Landau theory of second order transition.

Pseudowave-function $\psi$ acting as order parameter (in the normal phase $\psi = 0$, in the superconducting phase $\psi \neq 0$).

$\psi$ describes the superconducting electrons and their density

$$n_s = |\psi(r)|^2$$

The free energy density $f_s$ can be expanded in a series:

$$f_s = f_n + \alpha |\psi|^2 + \frac{\beta}{2} |\psi|^4 + \frac{1}{2m} |(-i\hbar \nabla - 2cA)\psi|^2 + \frac{|B|^2}{2\mu_0}$$

The order parameter and the vector potential are obtained by minimizing the Ginzburg-Landau formula with respect to $\psi$ and $A$. 
\[ f_s = f_n + \alpha \vert \psi \vert^2 + \frac{\beta}{2} \vert \psi \vert^4 + \frac{1}{2m} \left( -i\hbar \nabla - 2eA \right) \psi^2 + \frac{\hbar^2}{2\mu_0} \]

Let assume a situation without field and at an interface vacuum/superconductor.

Minimizing the free energy with respect to \( \psi \):

\[ \alpha \psi + \beta \vert \psi \vert^2 \psi - \frac{\hbar^2}{2m} \nabla^2 \psi = 0 \]

Taking into account that \( \psi(0) = 0 \) and that \( \psi(x \gg 0) = \psi_\infty \):

\[ \psi(x) = \psi_\infty \tanh \left( \frac{x}{\sqrt{2} \xi} \right) \]

with: \( \xi = \sqrt{\frac{\hbar^2}{2m \vert \alpha \vert}} \) \( \psi_\infty^2 = -\frac{\alpha}{\beta} \)

Order parameter

\[ \psi(0) = 0 \]

\[ \xi = \frac{\hbar \psi_F}{\pi \Delta} \]
Type I and Type II Superconductors

Type I

- Number of superelectrons
- Magnetic flux density
- Magnetic contribution $B^2/2 \mu_0$
- Condensation energy $-B_c^2/2 \mu_0$
- Total energy

Type II

- Number of superelectrons
- Magnetic flux density
- Magnetic contribution $B^2/2 \mu_0$
- Condensation energy $-B_c^2/2 \mu_0$
- Total energy

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Type I \((L < l)\)  

Type II \((L > l)\)

\[ B \]

\[-M\]

\[ H \]

\[ H_c(0) \]

\[ H_{c1}(0) \]

\[ H_{c2}(0) \]

Meissner phase

Vortex (Abrikosov) phase

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Vortex (Abrikosov) phase

Magnetic flux quantum
\( \phi_0 = \frac{\hbar}{2e} = 2.067 \cdot 10^{-15} \text{ Wb} \)

Bitter Decoration
Pb-4at%In rod, 1.1K, 195G
Muons and Field Distribution in Type II S.C.

Bishop et al., Scientific American 48 (1993)
Using Bulk Muon Spin Rotation to Study Superconducting Materials
Flux-line lattice (Abrikosov lattice)

elementary flux quantum
\[ \Phi_0 = \frac{h}{2e} = 2.067 \times 10^{-15} \text{ Vs} \]
Since the muon is a local probe, the SR relaxation function is given by the weighted sum of all oscillations:

\[ G(t) = \int f(B_\mu) \cos(\gamma_\mu B_\mu t) dB_\mu \]
SR a Type II Superconductor

\[ T > T_c \]

\[ \text{YBa}_2\text{Cu}_3\text{O}_{6.97} \]
\[ \text{Pümpin et al., Phys. Rev. B 42, 8019 (1990)} \]
SR a Type II Superconductor

$T < T_c$

$T > T_c$

$\text{YBa}_2\text{Cu}_3\text{O}_{6.97}$

Field Distribution in “Extreme” Type II S.C.

- Ginburg-Landau parameter

\[ \kappa \ll \frac{\lambda}{\xi} \ll 1. \]

- Large range of fields (up to \( B_{c^2/4} \)) where London model applies

- Vortex cores well separated and do not interact

- Vortex fields superimpose linearly

In an ideal vortex state the vectors \( \mathbf{a}_n \) form a periodic two-dimensional lattice. Therefore \([7-4]\) can be solved in Fourier space (k-space).

For an hexagonal lattice:

\[ a| = b - d, a \cdot b = \cos 60^\circ \]

Reciprocal vectors:

\[ \mathbf{a}^* = \frac{2\pi}{a} \mathbf{b} \times \mathbf{c} \]

\[ b^* = \frac{2\pi}{b} \mathbf{c} \times \mathbf{a} \]

\[ d^* = \frac{2\pi}{d} \mathbf{a} \times \mathbf{b} \]

\[ \mathbf{k}_{\min} = m \mathbf{a}^* - nb^* \]

(also hexagonal symmetry)

\[ \mathbf{B}(\mathbf{r}) = \sum_k b_k e^{i \mathbf{k} \cdot \mathbf{r}} \]

With Fourier components:

\[ b_k = \frac{1}{S} \int \mathbf{B}(\mathbf{r}) e^{-i \mathbf{k} \cdot \mathbf{r}} d^2 \mathbf{r} \]
Field distribution: \( \mathbf{B}(\mathbf{r}) \)?

\( \mathbf{B}(\mathbf{r}) \) must fulfill the modified London equation:

\[
\mathbf{B}(\mathbf{r}) - \lambda^2 \Delta \mathbf{B}(\mathbf{r}) = \phi_0 \sum_n \delta(\mathbf{r} - \mathbf{r}_n) \hat{\mathbf{z}}
\]

We expect a periodic magnetic field and therefore can use:

\[
\mathbf{B}(\mathbf{r}) = \sum_{\mathbf{K}} \mathbf{B}(\mathbf{K}) \exp(i\mathbf{K} \cdot \mathbf{r})
\]

with Fourier components:

\[
\mathbf{B}(\mathbf{K}) = \frac{1}{S} \int \mathbf{B}(\mathbf{r}) \exp(-i\mathbf{K} \cdot \mathbf{r}) d^2\mathbf{r}
\]

The modified London equation becomes (fields only along \( \hat{\mathbf{z}} \)):

\[
\sum_{\mathbf{K}} \left( \mathbf{B}(\mathbf{K}) + \lambda^2 K^2 \mathbf{B}(\mathbf{K}) \right) \exp(i\mathbf{K} \cdot \mathbf{r}) = N\phi_0 \sum_{\mathbf{K}} \exp(i\mathbf{K} \cdot \mathbf{r})
\]

and one finds:

\[
B_z(\mathbf{K}) = \frac{B}{1 + \lambda^2 K^2}
\]
By measuring the second moment of the field distribution (for example by µSR), we directly determine the London penetration.
Extract Information from the SR data

Field distribution depends on the microscopic parameters of superconductivity and

Theoretical models of the flux line lattice to fit the SR data

Structure, symmetry of the flux line lattice

Vortex motion

Characteristic lengths: magnetic penetration depth $\lambda$, radius of the vortex core $\xi$ - coherence length $\xi$

Classification scheme of superconductors

Theory

Maisuradze et al., (2008)

Pd-In alloy

Extract Information from the SR data

**Single crystals**
- Anisotropy field distribution

**Polycrystals or sintered sample**
- Disorder of pinning sites
- Strong smearing of the field distribution


$YBa_2Cu_3O_{6.97}$

$\lambda = 130 (10) \text{ nm}$

Sonier et al., PRL 83, 4156 (1999)

$YBa_2Cu_3O_{6.95}$

$\lambda = 150 (4) \text{ nm}$


Pd-In alloy

Zurab Guguchia
Extract Information from the SR data

\[ G(t) = \exp\left(-\frac{1}{2} \sigma^2 t^2\right) \times \cos(\gamma \mu \langle B_{\mu}^z \rangle t) \]

where: \( \sigma^2 = \gamma^2 \langle \Delta B_{\mu}^z \rangle^2 \)

**Ginzburg-Landau model**

\[ \langle \Delta B_{\mu}^z \rangle = 0.00371 \frac{\phi_0^2}{\lambda^4} \]

**London model**

\[ \lambda = \sqrt{\frac{m}{\mu_0 e^2 n_s}} \]

\[ \Rightarrow \sigma \propto \frac{1}{\lambda^2} \propto \frac{\mu_0 e^2}{m} n_s \]

Pümpin et al.,

A SR measurement of the second moment of the field distribution allows to determine the London penetration depth \( \lambda \).

The damping of a TF-SR spectrum is proportional to the super fluid density \( n_s \) (number of Cooper pairs).
Conventional superconductors have low $T_C$ and high superfluid density

Unconventional superconductors have relatively low superfluid density ("dilute superfluid")
Pairing Symmetry in Cuprates

Wave function of two electrons:
\[ \psi(r_1, s_1; r_2, s_2) = \psi(r_1, r_2) \chi(s_1, s_2) \]
where:
- \( \psi(r_1, r_2) \): space part
- \( \chi(s_1, s_2) \): spin part

\[ \chi = \frac{1}{\sqrt{2}} (|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle) \]

\[ \Rightarrow S = 0 \rightarrow \text{space part must be even.} \]
\[ \Rightarrow \text{s-wave (l = 0), d-wave (l = 2), etc...} \]

\[ \chi = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle, \ldots) \]

\[ \Rightarrow S = 1 \rightarrow \text{space part must be odd.} \]
\[ \Rightarrow \text{p-wave (l = 1), f-wave (l = 3), etc...} \]

Gap function: \( \Delta(k) \) has a lower symmetry than the Fermi surface

As the gap disappears along some directions of the Fermi surface (“nodes”), extremely-low-energy quasiparticles excitations (and therefore significant pair-breaking) may occur at very low temperature.
**s-wave Superconducting Gap**

- BCS conventional pairing: isotropic s-wave pairing

From $\mu$SR:

$$\sigma_{\mu} \propto \frac{1}{\chi^2} = \frac{\mu_0 e^2}{m} n_S$$

$$n_S(T) = n_S(0) \left(1 - \frac{2}{k_B T} \int_0^\infty f(\varepsilon, T)(1 - f(\varepsilon, T)) d\varepsilon \right)$$

and for an isotropic energy gap (s-wave):

$$n_S(T) = n_S(0) \left(1 - \sqrt{\frac{2\pi \Delta(0)}{k_B T}} \exp \left[ - \frac{\Delta(0)}{k_B T} \right] \right)$$

B. Mühlschlegel, Z. Phys. 155, 313 (1959)
**d-wave Pairing Symmetry**

\[ n_s(T) = n_s(0) \left( 1 - \frac{1}{\pi k_B T} \int_0^{2\pi} \int_0^\infty f(\epsilon, T) \left[ 1 - f(\epsilon, T) \right] d\phi d\epsilon \right) \]

with:

\[ f(\epsilon, T) = \left( 1 + \exp \left[ \sqrt{\epsilon^2 + [\Delta_s(T) \cos(2\phi)]^2 / k_B T} \right] \right)^{-1} \]

rembering that:

\[ \lambda = \frac{m}{\mu_0 e^2 n_s} \]

one gets: (for \( T \ll T_c \))

\[ n_s(T) \propto n_s(0) \left( 1 - 2C \frac{T}{\Delta_s(0)} \right) \]

\[ \lambda(T) \propto \lambda(0) \left( 1 + C \frac{T}{\Delta_s(0)} \right) \]

---

**Single crystal YBa\(_2\)Cu\(_3\)O\(_{6.95}\)**

J.E. Sonier et al, PRL 72, 744 (1994)
Multiband Superconductivity

ARPES on Ba$_{1-x}$K$_x$Fe$_2$As$_2$:

- Multiband superconductivity
- Nodeless gaps

$\Delta(k_x, k_y)$

$\lambda^{-2}(T) = \frac{\lambda^{-2}(T, \Delta_{0,1})}{\lambda^{-2}(0, \Delta_{0,1})} + \frac{\lambda^{-2}(T, \Delta_{0,2})}{\lambda^{-2}(0, \Delta_{0,2})}$


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\[ \rho_{\text{tot}}(T) = w\rho_{1}(T) + (1 - w)\rho_{2}(T) \]

\[ \rho_{i}(T) = \frac{\chi_{\alpha\beta}^{-2}(T, \Delta_{0,i})}{\chi_{\alpha\beta}^{-2}(0, \Delta_{0,i})}, \quad i = 1, 2 \]

\[ \Delta_{0,1} = 9.06(2)\text{ meV}, \quad \Delta_{0,2} = 1.50(2)\text{ meV}, \quad w = 0.51(2) \]

\[ \Delta_{0,1} = 1.1(3)\text{ meV}, \quad \Delta_{0,2} = 7.5(2)\text{ meV}, \quad \omega = 0.15(3) \]

Melting of the vortex lattice

[Diagram showing a phase diagram of a high-Tc type II superconductor, with regions for Meissner phase, vortex solid phase, vortex liquid phase, normal phase, and critical fields $B_{c1}(T)$ and $B_{c2}(T)$]
Vortex lattice melting

Lineshape asymmetry parameter $\alpha$

“skewness parameter”

$$\alpha = \frac{\langle (\Delta B)^3 \rangle^{1/3}}{\langle (\Delta B)^2 \rangle^{1/2}}$$
Vortex lattice melting

\[ \alpha = \frac{\langle (\Delta B)^3 \rangle^{1/3}}{\langle (\Delta B)^2 \rangle^{1/2}} \]

Using Low Energy Muon Spin Rotation to Study Superconducting Materials
Low Energy SR

Variable muon energy: 0 – 30 keV

Depth-sensitive local magnetic spin probe on the nm length scale

- Thin films
- Near-surface regions
- Multilayers
- Buried interfaces
Low Energy SR Apparatus

- Spin-rotator (E x B)
- Spin (E x B)
- Einzel lens (LN\textsubscript{2} cooled)
- moderator
- “surface” \(+\) beam, \(\sim 4\) MeV
- MCP detector
- Conical lens
- Start detector (10 nm C-foil)
- Einzel lens (LN\textsubscript{2} cooled)
- Sample cryostat e+ detectors

Ultra High Vacuum
\(\sim 10^{-10}\) mbar
Low Energy SR Apparatus

- Spin-rotator (E x B)
- Einzel lens (LN$_2$ cooled)
- Moderator

Moderator
- Source of low energy muons
Low Energy SR Apparatus

- Electrostatic mirror
- Spin-rotator (E x B)
- Einzel lens (LN$_2$ cooled)
- Moderator
- MCP detector
- "Surface" beam, ~4 MeV

Moderator

- 15 eV $\uparrow^+$
- 4 MeV $\uparrow^+$

- $<500$ nm
- $<100$ $\upmu$m
- s-Ne, s-Ar, Ag at 6K
- s-N$_2$

Efficiency $\sim 10^{-4} – 10^{-5}$

- Spin
- Start detector (10 nm C-foil)
- Conical lens
- Sample cryostat e+ detectors
- s-Ne, s- Ar, Ag at 6K
- s-N$_2$
Low Energy SR Apparatus

7.5 – 20 keV

Moderator

4 MeV

<500 nm <100 m
s-Ne, s-Ar, Ag at 6K
s-N₂

Efficiency ~ 10⁻⁴ – 10⁻⁵
Low Energy SR Apparatus

Electrostatic mirror
- Deflection of the low energy muons

- Spin-rotator (E x B)
- Einzel lens (LN$_2$ cooled)
- Moderator
- MCP detector
- Spin
- "surface" + beam, ~4 MeV

- Start detector (10 nm C-foil)
- Conical lens
- Sample cryostat
- e+ detectors
**Low Energy SR Apparatus**

- **Spin Rotator**: Rotates muon spin for different experimental conditions.

- **Components**:
  - Spin-rotator (E x B)
  - Einzel lens (LN$_2$ cooled)
  - Moderator
  - MCP detector
  - Electrostatic mirror
  - Conical lens
  - Start detector (10 nm C-foil)
  - Sample cryostat e+ detectors
  - E-Field
  - B-Field

- "Surface" ($^+$ beam, ~4 MeV)

---

Zurab Guguchia
Low Energy SR Apparatus

- Spin-rotator (E x B)
- Einzel lens (LN$_2$ cooled)
- Moderator
- MCP detector
- Electrostatic mirror
- "Surface" beam, ~4 MeV

Trigger detector

- Start of the LE-SR measurement
- Time-of-flight measurements
Low Energy SR Apparatus

Sample cryostat
- Deceleration and acceleration of the $^1+$
Low Energy SR Apparatus

Sample cryostat
- Deceleration and acceleration of the $\uparrow^+$
Example:
Magnetic field profile in type-I and type-II superconductors
Depth dependent $\mu$SR measurements

\[ B(z) = B_{ext} \exp(-z/\lambda) \]

\[ \omega_{\mu}(z) = \gamma_{\mu} B_{loc}(z) \]

More precise: use known implantation profile

\[ n(z, E) \]: muon implantation profile for a particular muon energy \( E \)

\[ \mu SR \text{ experiment } \Rightarrow \text{magnetic field probability distribution } p(B, E) \text{ sensed by the muons} \]

\[ n(z, E) \, dz = p(B, E) \, dB \]

\[ \int_0^z n(\zeta, E) \, d\zeta = \int_{B(z)}^{\infty} p(\beta, E) \, d\beta \]

→ Magnetic field profile \( B(z) \) over nm scale

Direct measurement of the magnetic penetration depth

\[ B(z) = B_{\text{ext}} \exp(-z/\lambda) \]

SR experiment \( p(B) \) at the muon site

\( p(B) \) \( B(z) \)


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Direct measurement of $\lambda$ in a YBa$_2$Cu$_3$O$_7$ film

Summary
**Muon Spin Rotation / Relaxation (SR)**

### Magnetism:
- **Local probe**
  - Magnetic volume fraction
- **SR frequency**
  - Magnetic order parameter \( (10^{-3} - 10^{-4} \, \text{B}) \)
  - Temperature dependence
- **SR relaxation rate**
  - Homogeneity of magnetism
- **Magnetic fluctuations**
  - Time window: \( 10^5 - 10^9 \, \text{Hz} \)

### Superconductivity:
- **Field distribution of vortex lattice**
  - Penetration depth
  - Coherence length
  - Vortex dynamics
- **Absolute determination of penetration depth** \( H(0) \)
- **Temperature dependency of**
  - Penetration depth, \( 4 <\langle \mathcal{B}^2 \rangle>^2 \, \mathcal{I}^2 \)
  - Superfluid density \( n_s/m^* <\langle \mathcal{B}^2 \rangle> \, \mathcal{I} \)
  - Symmetry of the SC gap function

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