The Fifth International Conference on Applications of Mathematics and Informatics in Natural Sciences and Engineering

(AMINSE5, Tbilisi, Georgia, 2020+1, Virtual)

Dedicated to the 25th Anniversary of Foundation of
Tbilisi International Centre of Mathematics and Informatics (TICMI)
http://www.viam.science.tsu.ge/aminse2020/

BOOK OF ABSTRACTS

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TBILISI, GEORGIA

ORGANIZERS:

- I. Javakhishvili Tbilisi State University
  - I. Vekua Institute of Applied Mathematics
  - Faculty of Exact and Natural Sciences
- Georgian Mechanical Union
  - Georgian National Committee of Theoretical and Applied Mechanics
- Tbilisi International Centre of Mathematics and Informatics

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Conference web-page: http://www.viam.science.tsu.ge/aminse2020/

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25 Years of TICMI
Tbilisi International Centre in Mathematics and Informatics (1995-2020)

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In 1995 TICMI (http://www.viam.science.tsu.ge/ticmi) was founded by initiation of Prof. George Jaiani with the support of EMS (European Mathematical Society) and personally of its first President Prof. Friedrich Hirzebruch, who beforehand on 16.11.1994 invited G. Jaiani to Bonn at Max-Planck Institute in order to discuss aims and prospective role of the centre in Caucasian independent republic of Georgia. He was the first who used the acronym TICMI with words "TICMI is a good idea" during the above mentioned discussions and instruct EMS secretary of that time Prof. E.C. Lance to carry out the corresponding actions on the part of EMS.

TICMI is based in the I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University.

STATUTE

1. The Tbilisi International Centre of Mathematics and Informatics (later called "Centre") has been founded in 1995 under the aegis of the Academy of Natural Sciences of Georgia (later "Academy"). Based in the I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University (later "Institute"), it is supported by the Georgian Mathematical Union and the European Mathematical Society.

2. The aims of the Centre are, first, to help young scientists of the countries of the Black Sea Basin, to improve their professional skills and, second, to promote the exchange of scientific information worldwide. This will be achieved by Lecture Series and Summer Schools devoted to topics determined by the immediate interests of the Centre. The goal of TICMI is to be a scientific center for Georgia, Turkey, Armenia, Azerbaijan

3. The work of the Centre is guided by an International Scientific Committee (later called "Committee"), consisting of seven persons. Four are elected for a period of four years by the Scientific Council of the Institute; they represent different interests and at least one among them is a member of the Institute. Their candidatures are approved by the Presidium of the Georgian Mathematical Union and by the Department of Mathematics and Informatics of the Academy. The three others are appointed for the same period by the Executive Committee of the European Mathematical Society.
    a. The Committee constructs long-term and annual plans for the Centre. It identifies themes, selects lecturers, compiles lists of participants and decides on the awarding of grants (up to 20 percent of participants), and discusses current problems.
    b. The Committee meets once a year to deal with the relevant business placed before it. A Committee decision requires a vote of approval from at least four members of the Committee.

c. The Committee has the Chairman whose candidature is approved by the Presidium of the Academy on the recommendation of the Director of the Institute.
d. The Chairman of the Committee, working within the decisions of the Committee directs the work of the Centre. He is supported by business manager and technical secretary, both chosen from among the Institute members.

4. The Committee is accountable to the Presidium of the Academy. It is required to provide annual report of its work to the Georgian Mathematical Union and European Mathematical Society.

5. The Centre is not an autonomous unit within the Institute. Its interests are governed and protected by the Institute and Academy.

6. The financial activities of the Centre are controlled by the Finance Office of the Institute.

Considered and Approved at

the Sitting of the Department of Mathematics and Informatics of the Academy of Natural Sciences of Georgia on January 11, 1995 (record # 1);

the Sitting of the Scientific Council of I.Vekua Institute of Applied Mathematics of I.Javakhishvili Tbilisi State University on January 25, 1995 (record # 1).

The first International Scientific Committee (ISC) of TICMI

G. Jaiani (Chairman), Tbilisi, I.Vekua Institute of Applied Mathematics, I. Javakhishvili Tbilisi State University (TSU),
J.-Cl. Hausmann, Geneve, Universite de Geneve,
E. Khmaladze, Tbilisi, A. Razmadze Mathematical Institute of the Georgian Academy of Sciences (GAS),
D. Natroshvili, Tbilisi, Georgian Technical University,
G.F. Roach, Glasgow, Strathclyde University,
V. Sollonnikof, Petersburg, St. Petersburg Department of Steklov Institute of Mathematics of the Russian Academy of Sciences
M. Tsuladze, Tbilisi, N. Muskhelishvili Institute of Computational Mathematics of the GAS.

Among of them *J.-Cl. Hausmann, G.F. Roach*, *V. Sollonnikof* according to the Statute of TICMI, were nominated by the Executive Committee (EC) of EMS.

In different times the members of ISC of TICMI were:

J. Sanikidze, Tbilisi, N. Muskhelishvili Institute of Computational Mathematics of the GAS,
T. Shervashidze, Tbilisi, A. Razmadze Mathematical Institute of the GAS,
D. Cioranescu, Paris, Laboratoire d'Analyse Numerique, Universite Paris V,
M. Giaquinta, Pisa, Scuola Normale Superiore,
P. Freitas, Lisbon, University of Lisbon,
O. Gil-Medrano, Valencia, Universidad de Valencia

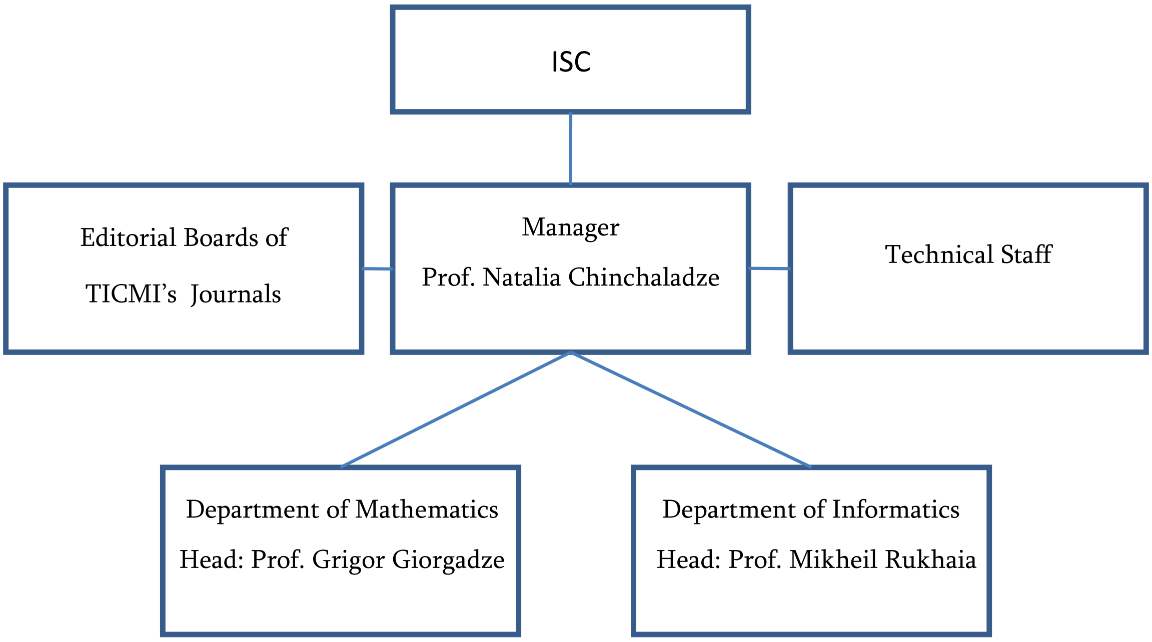
Among of them *D. Cioranescu*, *M. Giaquinta*, *P. Freitas*, *O. Gil-Medrano* were nominated by EC of EMS.

At present ISC of TICMI looks like

L. Beznea, Bucharest, Simion Stoilow Institute of Mathematics of the Romanian Academy
A. Fialowski, Budapest, Institute of Mathematics, Pazmany Peter setany 1/C,
G. Jaiani (Chairman),Tbilisi, I.Vekua Institute of Applied Mathematics, TSU,
V. Kvaratskhelia, Tbilisi, N. Muskhelishvili Institute of Computational Mathematics of the Georgian Technical University,
A. Meskhi, Tbilisi, A. Razmadze Mathematical Institute of the TSU,
D. Natroshvili, Tbilisi, Georgian Technical University,
E. Shargorodsky, London, King's College London.

Among of them *L. Beznea*, *A. Fialowski*, *E. Shargorodsky* were nominated by EC of EMS.

Present Structure of TICMI



Since foundation TICMI has organized advanced courses (with Minisymposia) and workshops for young scientists and Students. In different times as lecturers (besides the Georgian scientists) were invited: *Wolfgang L. Wendland* (University Stuttgart, Germany), *B.-Wolfgang Schulze* (University of Potsdam, Germany), *Veronique Lods, Gerard Tronel* (Universite P. et M.Curie, France), *Paolo E. Ricci* (University of Rome, "La Sapienca", Italy), *Alois Kufner* (Mathematical Institute, Academy of Sciences, Czech Republic), *Diego Caratelli* (University of Rome, "La Sapienza", Italy), *Alberto Cialdea* (University of Potenza, Italy), *Flavia Lanzara* (University of Rome, "La Sapienza", Italy), *Paolo Podio-Guidugli* (Università di Roma 2, TorVergata, Italy), *Alemdar Hasanoglu* (Izmir University, Izmir, Turkey), *Wolfgang H. Mueller* (Technical University of Berlin, Germany), *Holm Altenbach* (Otto-von-Guericke-University Magdeburg, Faculty of Mechanics, Germany), *Alice Fialowski* (Eotvos Lorand University, Institute of Mathematics, Hungary), *Reinhold Kienzler* (University of Bremen, Germany), *Ralf Schindler* (University of Münster, Germany), *Ayech Benjeddou* (Institut Supérieur de Mécanique de Paris & Université de Technologie de Compiègne /Centre National de la Recherche Scientifique FRE 2012 ROBERVAL, France), *Bernadette Miara* (Université Paris-Est Marne la Vallée, France).

List of Advanced Courses with Minisymposia

- Advanced Courses on Mathematical Models of Piezoelectric Solids and Related Problems, 2019 (Number of participants 34)
- II Advance Courses in Applied Mathematics, 2015 (Number of participants 34)
- Advance Courses in Applied Mathematics, 2014 (Number of participants 29)
- Advanced Courses on Boundary Value Problems for Partial Differential Equations, 2010 (Number of participants 23)
- Advanced Courses on Methods of Effective and Explicit Solving of Boundary Value Problems, 2009 (Number of participants 24)
- Advanced Course on Informatics, 1999 (Number of participants 16)
- Advanced Course on Integral Operators and Related Problems, 1999 (Number of participants 34)
- Advanced Course on Function Spaces and Applications, 1999 (Number of participants 31)
- Advanced Course on Pseudodifferential Operators, 1998 (Number of participants 23)
- Advanced Course on Theory of Elasticity 2, 1997 (Number of participants 26)
- Advanced Course on Theory of Elasticity 1, 1996 (Number of participants 25)

List of Workshops

- Workshop in Discrete Mathematics, 2017 (Number of participants 20)
- Workshop in Discrete Mathematics, 2016 (Number of participants 17)
- Workshop on Applications of Mathematics to Biology and Medicine, 2011-2 (Number of participants 14)
- Workshop on 1D Nanostructures - Theory and Technology, 2011 (Number of participants 19)
- Workshop on Mathematical Methods in Natural Sciences, 2006 (Number of participants 12)
- Workshop on Degenerate Partial Differential Equations, Weighted Spaces, and Applications to Mechanics, 2005 (Number of participants 30)
- Workshop on Application of Mathematical Methods in Natural Sciences, 2004 (Number of participants 20)
- Workshop on Orthogonal Polynomials and Their Applications, 2003 (Number of participants 12)
- Workshop on Application of Mathematical Methods in Natural Sciences, 2002 (Number of participants 39)
- Workshop on Mathematical Models for Elastic Cusped Plates and Bars, 2001 (Number of participants 30)
- French-Georgian Workshop in Applied Mathematics, 2000 (Number of participants 27)
- TICMI Workshop 1, 1998 (Number of participants 25)

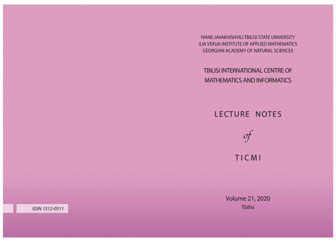
TICMI was co-organizer of the International Conferences

- XI Annual Meeting of the Georgian Mechanical Union, 2020
- The Fourth International Conference on “Application of Mathematics and Informatics in Natural Sciences and Engineering”, 2019
- X Annual Meeting of the Georgian Mechanical Union, 2019
- The Third International Conference on "Modern Problems in Applied Mathematics", 2018
- IX Annual Meeting of the Georgian Mechanical Union, 2018
- The Third International Conference on “Application of Mathematics and Informatics in Natural Sciences and Engineering”, 2017
- VIII Annual Meeting of the Georgian Mechanical Union, 2017
- VI Annual Meeting of the Georgian Mechanical Union, 2015

- V Annual Meeting of the Georgian Mechanical Union, 2014
- IV Annual Meeting of the Georgian Mechanical Union, 2013
- International Conference on Fourier Analysis and Approximation Theory, 2013
- Second International Conference on "Modern Problems in Applied Mathematics", 2013
- III Annual Meeting of the Georgian Mechanical Union, 2012
- The First International Conference on "Modern Problems in Applied Mathematics", 2008
- IUTAM Symposium on Shells, Plates, and Beams, 2007
- ISAAC Conference on Complex Analysis, Partial Differential Equations, and Mechanics of Continua, 2007

TICMI publishes two international scientific journals

-"Bulletin of TICMI", editor G. Jaiani, founded in 1997
http://www.viam.science.tsu.ge/ticmi/bulletin/bulletin



-"Lecture Notes of TICMI", editor G. Jaiani, founded in 2000
http://www.viam.science.tsu.ge/others/ticmi/lnt/lecturen.htm

Both the journals are indexed in Mathematical Reviews/MathSciNet, Scopus, and Zentralblatt MATH/Mathematics Abstracts.

Rheological Modeling in Solid Mechanics from the Beginning up to Now

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The method of rheological modeling was developed approximately 100 years ago for a better modelling of complex material behavior. There were many contributions in that time connecting theory and experiments. The first state of the art report was the monograph of Marcus Reiner [1]. Heinrich Hencky suggested a general continuum mechanics based approach in the 20th of the last century.

Vladimir Palmov published his pioneering book in this field in 1976, in 1998 translated into English [2]. Later he discussed several approaches and introduced models for isotropic materials, but also for anisotropic, and finally extended to large strains.

The last part of the lecture is devoted to some actual applications.

References

[1] M. Reiner. Deformation, Strain and Flow: an Elementary Introduction to Rheology. H. K. Lewis, 1960.

[2] V. Palmov. Vibrations of Elasto-Plastic Bodies. Springer, 1998.

On One Nonparametric Estimate of Poisson Regression Function

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The limiting distribution of the integral square deviation of kernel-type nonparametric estimator of Poisson regression function is established. The test of the hypothesis testing about Poisson regression function is constructed. The question of consistency of the constructed test is studied. The power asymptotic of the constructed test is also studied for certain types of close alternatives.

Stress and Strength Analysis of a Circular Hole in an Orthotropic Plate with Finite Width

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Open circular holes occur in a multitude of structural situations, for instance in bolted joint connections between plates. The presence of such holes in many cases causes stress concentrations depending on the applied load, the given material anisotropy as well as the hole size relatively to the outer finite plate dimensions. For safe design and optimal material use sufficiently reliable structural analysis tools are needed. In the present contribution the stress field of the finite-width open-hole problem is determined for the case of uniaxial tension by means of complex potential method. The given boundary value problem is solved by starting with the infinite domain problem and supplementing it with auxiliary potentials enabling to take into account traction-free edges. The results are validated by comparative finite element analyses with excellent agreement. Next the effective strength of the given open-hole situations is addressed and predicted by means of the Theory of Critical Distances and by means of Finite Fracture Mechanics. These assessment concepts are compared to each other and to given experimental findings.

Research of a Three-Dimensional Dynamic System Describing the Process of Assimilation

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The work proposes a nonlinear mathematical model describing the process of two-level assimilation, taking into account the quadratic terms of self-limiting growth of three populations speaking three languages, significantly different in dissemination.

Two-level assimilation process considered when in one large region the population speaking the most common language assimilates both the population speaking the second fairly common language and the population speaking the third less common language (small range of language dissemination). In turn, the population speaking the second language, which is quite common, assimilates the population speaking the less common third language. Thus, the population speaking the third less common language is in a situation of bilateral assimilation.

The work assumes that the process of assimilation develops due to numerous direct or remote (electronic communication) mutual meetings between representatives of the population, who consider one of these three languages to be their native language.

In the special case, when the coefficients of the mathematical model are constants, the first integral of a nonlinear three-dimensional dynamic system is found, which in the phase space of solutions is a hyperbolic paraboloid. Using the first integral, the three-dimensional dynamic system is reduced to two-dimensional and, taking into account Bendixon's criterion, the theorem about the existence of closed integral trajectory solutions in the first quarter of the phase plane has been proved. Thus, conditions were found for model coefficients at which there is no complete assimilation of the population speaking on the third, less common language.

Functional Dissipativity of Second Order Differential Operators with Complex Coefficients

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In this talk I will present some recent results obtained with Vladimir Maz'ya in [1]. They concern the Dirichlet problem for the second order differential operator $E = \nabla(A\nabla)$, where A is a matrix with complex valued L^∞ entries. We have introduced the concept of dissipativity of E with respect to a given function $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$. This means that

$$Re \int_\Omega \langle A\nabla u, \nabla(\varphi(|u|)\, u)\rangle \, dx \geq 0$$

for any complex valued $u \in \mathring{H}^1(\Omega)$ such that $\varphi(|u|)\, u \in \mathring{H}^1(\Omega)$, Ω being a domain in $\mathbb{R}^N$. Under the assumption that the $Im\, A$ is symmetric, we have found the necessary and sufficient conditions for the functional dissipativity of E.

References

[1] Cialdea A., Maz'ya V., (2021) Criterion for the functional dissipativity of second order differential operators with complex coefficients, *Nonlinear Analysis*, (206) 112215, DOI: 10.1016/j.na.2020.112215.

Numerical Simulation of Glacier Adaptation to Regional Climate Alteration

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Glaciers are one of the main indicators of current climate change, as the interaction between a glacier and climate is a complex non-linear process. Climate change characterized by fluctuations in the balance of radiation energy in the lower troposphere, which determines the process of fluctuating glaciers (melting of thickness). In addition, the abundant glacial ensemble with its special properties contributes to some extent to climate variation. Therefore, in mathematical modeling of the dynamics of glaciers, it is not easy to take into account these nonlinear processes completely.

In this article, a two-dimensional mathematical model of the dynamics of changes in the thickness of glaciers in the Caucasus has been developed, based on the integration of nonlinear partial differential equations, which in turn is ensured by a change in the equilibrium mass of the glacier due to direct solar radiation. The configuration of the upper surface of the glacier is predicted by solving the continuity equation. A scheme, similar to the Lax-Wendroff scheme, is used to solve numerically the nonlinear PDE model. Some typical problems of mathematical and numerical modeling of glaciers are discussed. For the first time, the process of melting of some glaciers in the Caucasus has been assessed using mathematical modeling. Some simulation results are presented and analyzed.

Acknowledgement: This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSF) (Grant Number FR17_548).

Convolution Equations on the Abelian Group $\mathcal{A}(-1,1)$ and Their Applications

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The interval $\mathcal{J} := [-1,1]$ turnes into an Abelian group $\mathcal{A}(-1,1)$ if endowed with the group operation $x +_{\mathcal{J}} y := \dfrac{x+y}{1+xy}$, $x, y \in \mathcal{J}$. The invariant Haar measure is $d_{\mathcal{J}}x := \dfrac{dx}{1-x^2}$ and the Fourier transformation is

$$(\mathcal{F}_{\mathcal{J}}v)(\xi) := \int_{-1}^{1}\left(\frac{1-y}{1+y}\right)^{i\xi}\frac{v(y)dy}{1-y^2} = \int_{-1}^{1}\left(\frac{1-y}{1+y}\right)^{i\xi} v(y)d_{\mathcal{J}}y, \qquad \xi \in \mathbb{R}. \tag{1}$$

These tools allow to solve exactly the following convolution integro-differential equations on this group

$$\sum_{k=0}^{n}\left[a_m\mathfrak{D}^k u(x) - b_m\int_{-1}^{1}\mathcal{K}_m\left(\frac{x-y}{1-xy}\right)\mathfrak{D}^k u(y)d_{\mathcal{J}}y\right] = f(x), \qquad x \in \mathcal{J}, \tag{2}$$

where $\mathcal{K}_m \in \mathbb{L}_1(\mathcal{J}, d_{\mathcal{J}}x)$, $a_m, b_m \in \mathbb{C}$ are complex numbers (m=0,1,...,m) and $\mathfrak{D}u(x) := -(1-x^2)\dfrac{d}{dx}u(x)$ is the natural derivative on the group $\mathcal{A}(-1,1)$.

To the class of convolution equations (2) belong celebrated equations with important applications- the Prandtl equation

$$\boldsymbol{P}u(x) = \frac{c_0u(x)}{1-x^2} + \frac{c_1}{\pi i}\int_{-1}^{1}\frac{u'(y)dy}{y-x} = f(x), \qquad x \in \mathcal{J} \tag{3}$$

the Singular Tricomi equation

$$\boldsymbol{T}v(x) = c_0v(x) + \frac{c_1}{\pi i}\int_{-1}^{1}\frac{v(y)dy}{y-x} + \frac{c_2}{\pi i}\int_{-1}^{1}\frac{v(y)dy}{1-xy} = g(x), \qquad x \in \mathcal{J} \tag{4}$$

and Lavrentjev-Bitsadze equation

$$\boldsymbol{LB}\varphi(x) = c_0\varphi(x) + \frac{c_1}{\pi}\int_{0}^{1}\left[\frac{1}{y-x} + \frac{1-2y}{x+y-2xy}\right]\varphi(y)dy = h(x), \qquad x \in \mathcal{I} := (0,1). \tag{5}$$

These equations have ample of applications in Mechanics and Mathematical Physics and were investigated by many authors. Equations (3) and (4) were solved by V. E. Petrov in [1] in the general Banach spaceless setting, while in [3] equation (5) was investigated in the Bessel potential space setting $\widetilde{\mathbb{H}}^s(\mathcal{J})$, $-1 \leqslant s \leqslant 1$, which was defined as the image of the corresponding Bessel potential space $\mathbb{H}^s(\mathbb{R})$ on the axes under the diffeomorphism of $\mathcal{J}$ and $\mathbb{R}$. N. Shavlakadze also solved Prandtl equation (3) in his recent work. These authors use the Fourier transformation (1), defined as the equivalent transformation to the classical Fourier transformation on the real axes under the diffeomorphism of $\mathcal{J}$ to $\mathbb{R}$ and its inverse: $t = \dfrac{1}{2}\ln\dfrac{1-x}{1+x}$, $x = -\tanh t$, $x \in \mathcal{J}$, $t \in \mathbb{R}$.

We solve equations (2)-(5) in the full scale of $\mathbb{L}_p(\mathcal{J}, d_{\mathcal{J}}x)$-based Bessel potential spaces $\mathbb{H}^s_p(\mathcal{J}, d_{\mathcal{J}}x)$ for $-\infty < s < \infty$, $1 < p < \infty$. For integer $s = m = 1, 2, \ldots$ $\mathbb{H}^m_p(\mathcal{J}, d_{\mathcal{J}}x)$ coincides with the Sobolev space $\mathbb{W}^m_p(\mathcal{J}, d_{\mathcal{J}}x)$, consisting of functions $\varphi \in \mathbb{L}_p(\mathcal{J}, d_{\mathcal{J}}x)$ which have the distributional derivatives $\mathfrak{D}^k\varphi \in \mathbb{L}_p(\mathcal{J}, d_{\mathcal{J}}x)$ for all $k = 1, 2, \ldots, m$.

References

[1] Petrov V. E.,*J. Math. Sci.* **132** (2006), no. 4, 451481.

[2] Petrov V. E., *Dokl. Math.* **74** (2006), no. 3, 901905.

[3] Suslina T.A., Petrov V.E., https://arxiv.org/abs/2008.06715, 17 pages, 2020.

On the Zeroes and Extrema of Generalised Clausen Functions

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Generalised Clausen functions arise in many areas of research in mathematics and physics. Despite their importance in such different areas as Hodge theory, number theory and quantum field theories, many properties still remain to be investigated. In particular, there seems to be no account on the location of their zeroes and extreme values available in the literature. However, it is well known that generalised Clausen functions may be expressed in terms of Bernoulli polynomials in some special cases. We will take advantage of this relationship in order to extend known results about Bernoulli polynomials to the generalised Clausen functions.

On Lie Superalgebras and Their Moduli Spaces

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I will introduce the notion of Lie superalgebra and talk about its importance in physics. In a given dimension they show a beautiful geometric picture. We conjecture that their moduli space consists of some families and some singleton elements. I will demonstrate it in low dimension and show how deformation theory helps to uniquely decompose the moduli space into different strata. I will demonstrate the moduli space with pictures.

Boundary Value Problems on Corner Manifolds

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Boundary value problems (BVPs) on corner manifolds suggest applying Boutet de Monvels calculus of operators on a smooth manifold with the transmission property at the boundary to the corresponding case with corner singularities. We give an outline of such an approach for regular singularities, characterized by transversal intersections of the strata close to the singular points. Another aspect are symbol hierarchies of the operators, determined by the strata of the configuration, similarly to those in BVPs. Singularities of first order concern conical points, or edges. Smoothness up to the boundary corresponds to singularity of order zero. This talk is aimed to introduce an analog of Boutet de Monvel's calculus for the case when the underlying configuration is a manifold with edge and boundary. The motivation comes from applications to elliptic BVPs in such a singular situation.

Singular Integrals and Boundary Value Problems

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The goal of this lecture is to present recent results of the speaker concerning mapping properties of various singular integral operators in grand function spaces and applications to the boundary value problems of analytic functions (vectors). More precisely, we plan:

- to establish the necessary and sufficient conditions both for the weights and curves ensuring the boundedness of Calderón commutators and Cauchy singular integral operators in new grand function spaces.
- to describe the mapping properties of generalized Cauchy singular integral arising in I. Vekua's theory of generalized analytic functions.
- solution of Riemann boundary value problem with oscillating coefficient and free term from grand variable function spaces.

Acknowledgement: This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSF) (Grant Number FR-2499).

Fast Computation of Elastic and Hydrodynamic Potentials Using Approximate Approximations

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We propose fast cubature formulas for the elastic and hydrodynamic potentials based on the approximate approximation of the densities with Gaussian and related functions. For densities with separated representation, we derive a tensor product representation of the integral operator which admits efficient cubature procedures. We obtain high order approximations up to a small saturation error, which is negligible in computations. Results of numerical experiments which show approximation order $O(h^{2M})$, M=1,2,3,4, are provided. This talk is based on a joint work with Vladimir Mazya and Gunther Schmidt.

Einstein Spacetimes with Low Regularity or Slow Decay

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I will present recent advances on spacetimes satisfying the Einstein equations. For dealing with the so-called colliding gravitational wave problem in general relativity, in collaboration with B. Le Floch (Paris) and G. Veneziano (Geneva) the existence of a large class of Lorentzian manifolds containing singularity hypersurfaces and satisfying suitable junction conditions. We also introduce the notion of a cyclic spacetime (also called a multiverse) consisting of spacetime domains bounded by spacelike or timelike singularity hypersurfaces, across which our scattering map is applied. In another direction, the Seed-to-Solution Method (in a joint work with T. Nguyen, Paris) is a novel approach for analyzing Einsteins constraint equations and provides us with asymptotically Euclidean manifolds with prescribed asymptotics. We encompass metrics with the weakest possible decay (infinite ADM mass) or the strongest possible decay (Schwarzschild behavior). Motivated by Carlotto and Schoens pioneering work on the optimal localization problem, we establish sharp decay estimates for the Einstein operator and solve what we call the asymptotic localization problem.

Non-local Contact Problem for Linear Partial Differential Equations of Parabolic Type with Constant and Variable Coefficients

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The present work is devoted to the formulation and investigation of a non-local contact problem for a parabolic-type linear differential equation with partial derivatives.

In the first part of the work, the linear parabolic equation with constant coefficients is considered. To solve a non-local contact problem, the variable separation method (Fourier method) is used. Analytic solutions are built for this problem.

One then elaborates on the non-local contact problem for parabolic equations with variable coefficients. Using the iterative method, the existence and uniqueness of the classical solution to the problem is proved. The proof of the existence and uniqueness of the solution is based on the use of the generalized Harnack theorem, which also is valid for linear differential equations with partial derivatives of parabolic type. The effectiveness of the method is confirmed by numerical calculations.

On Factorization of Monoids

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Given a monoid M, a *congruence* on a left M-set X (or, a (*left*) M-*congruence on* X), is an equivalence relation $\wp \subseteq X \times X$ on A such that $(x, x') \in \wp$ implies $(mx, mx') \in \wp$ for all $x, x' \in X$ and $m \in M$. (Similarly one defines a *right* M-*congruence* on a right A-set Y.) The $\wp$-equivalence class of an element $x \in X$ is denoted by $[x]_\wp$. In the special case of the left (resp. right) M-set (M, m_M), we write $\mathcal{C}^l(M)$ (resp. $\mathcal{C}^r(M)$) for the corresponding set of left (resp. right) congruences on it. A *transversal* of a congruence ϱ on a left (resp. right) M-set X is a set $T \subseteq X$, such that T consists of exactly one representative of every equivalence class of ϱ.

A monoid M is said to be *factorizable* if it contains two submonoids M_1 and M_2 such that the multiplication map $M_1 \times M_2 \to M$, $(m_1, m_2) \longmapsto m_1 m_2$ is bijective. The couple (M_1, M_2) is called a *factorization* of M.

The goal of our talk is to prove the following

Theorem. *For any monoid M, the assignment*

$$(\alpha, \beta) \longmapsto ([1_M]_\alpha, [1_M]_\beta)$$

yields a bijection between the set of factorizations of M and the set of pairs $(\alpha, \beta) \in \mathcal{C}^l(M) \times \mathcal{C}^r(M)$ such that

1. $\alpha \cap \beta = \triangle_M$,
2. $[1_M]_\beta$ *is a transversal of* α,
3. $[1_M]_\alpha$ *is a transversal of* β.

Stochastic Integral Representation of Past-Dependent Non-Smooth Brownian Functionals

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The stochastic integral representation theorem (also known as martingale representation theorem) states that any square-integrable Brownian functional is equal to a stochastic integral with respect to Brownian motion. The first proof of the martingale representation theorem was implicitly provided by Ito (1951). One of the pioneering works on explicit descriptions of the integrand belongs to Clark (1970). In this direction, significant results were obtained by Haussmann (1979), Ocone (1984), Ocone and Karatzas (1991) and Karatzas, Ocone and Li (1991), Shiryaev, Yor and Graversen (2003, 2006), Renaud and Remillard (2006). In many cases, one can determine the form of the stochastic integral representation using the Malliavin calculus if the functional is Malliavin differentiable [1]. Later, it turned out that the requirement of smoothness of the functional can be weakened by the requirement of smoothness only of its conditional mathematical expectation [2].

We consider stochastically (in the sense of Malliavin) non-smooth path-dependent Brownian functionals (therefore, one cannot use the well-known Clark-Ocone formula [1] and propose some method for obtaining a constructive representation. In addition, the class under consideration includes functionals for which even the conditional mathematical expectation isn't stochastically smooth and, consequently, the generalization of the Clark-Ocone formula proposed by Glonti-Purtukhia [2] is inapplicable to them. The results obtained can be used to construct a hedging strategy in various European options with corresponding payoff functions.

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Recent Results on Differential Models of Cell Movements on Heterogeneous Structures

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The present talk is motivated by the modelling of the mechanisms observed in laboratory experiments made on microfluid chip, where quantitative data on the dynamics of multiple cell species and interactions between tumor and immune cells are available. For this reason, we developed a tumor-immune spatial model to describe the behavior of immune cells and the concentration of chemicals in cancer-on-chip environment. Indeed, the model is based on coupling between models for cell movemente to diffusion equations for chemical gradient emanated by tumor cells and the particle model for the immune cells trajectories. Our effort is devoted to the development of a simulation tool that is able to reproduce the chemotactic movement and the interactions between different cell species living in a microfluidic chip environment and in an efficient and trustable tool for the validation of the model through suitable calibration techniques. Several tests are presented to show the effectiveness of our strategy.

Application of the Theory of Pseudodifferential Equations in the Multi-Field Mixed Problems for Composed Elastic Structures

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We investigate multi-field mixed dynamical problems for complex multi-layer elastic anisotropic structures containing interfacial cracks when in different adjacent components of the composed body different refined models of elasticity theory are considered. In particular, we analyse the case when we have the generalized thermo-electro-magneto elasticity model (GTEME model) in one region of the composed body and the generalized thermo-elasticity model (GTE model) in another adjacent region. Both models are associated with Green-Lindsay's model [1], [2]. This type of mechanical problem mathematically is described by systems of partial differential equations with appropriate boundary-transmission and initial conditions. In the GTEME model part we have six dimensional unknown physical field (three components of the displacement vector, electric potential function, magnetic potential function, and temperature distribution function), while in the GTE model part we have four dimensional unknown physical field (three components of the displacement vector and temperature distribution function). The diversity in dimensions of the interacting physical fields complicates mathematical formulation and analysis of the corresponding initial-boundary-transmission problems. We apply the Laplace transform technique, the potential method and the theory of pseudodifferential equations to prove uniqueness and existence theorems of solutions to different type basic and mixed initial-boundary-transmission problems in appropriate Sobolev spaces. We analyse the smoothness properties of solutions and establish asymptotic behaviour of the first derivatives near the exceptional curves (the crack edges and curves where different boundary conditions collide). It is shown that smoothness of solutions and stress singularity exponents at the exceptional curves essentially depend on the material parameters of the composed body. Moreover, we describe an efficient algorithm for evaluating the stress singularity exponents.

This is a joint work with Tengiz Buchukuri and Otar Chkadua.

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Some Boundary Value Problems for Maxwell Type System of Partial Differential Equations

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Some boundary value problems for Maxwell type system of partial differential equations are considered. Necessary and sufficient conditions, imposed on the coefficients involved in the boundary conditions, ensuring well-posedness of the problem are found. It is shown what type of violation of the correctness of the problem occurs when these conditions are not fulfilled. It is also shown what kind of changes in the initial conditions should be done to make the problem well-posed. In the case of well-posed problems, the solutions are constructed explicitly.

Computing Matrix Roots by 2nd Kind Pseudo-Chebyshev Functions and Dunford-Taylor Integral

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After recalling a technique for computing matrix roots using the 2nd kind pseudo-Chebyshev functions [1, 2, 3, 4], a general method, based on the representation formula for the resolvent [5] of a complex $r \times r$ matrix $\mathcal{A}$ and on Dunford-Taylor's integral (also ascribed to Riesz-Fantappiè) is exploited for computing the nth roots of $\mathcal{A}$. Only the invariants of the considered matrix are used. The knowledge of eigenvalues is not necessary. Several worked examples have been made by the first author, by using the computer algebra system Mathematica©, checking the validity of this procedure even for higher order non-singular random matrices [6].

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